1.3 Solving Exponential Equations

Sept. 5

Express 16 as a power of 2 $\Rightarrow$ this means $2^? = 16$

$\therefore \; ? = 4$

$\therefore$ 16 as a power of 2 is 2^4

Solve: $3^x = 3^{17}$ for two powers with the **same base** to be equal, the exponents must be equal.

$\therefore \; x = 17$

Solve: $3^x = 9$ need the bases to be the same. Express 9 as a power of 3.

$\underbrace{3^x = 3^2}$ → this is still "9" – just changed the look.

$\therefore x = 2$ ← for powers, with the same base, to be equal, their exponents must be equal.

Solve: $7^x = 343$ ← rewrite 343 so the bases are the same $\Rightarrow$ $343 = 7^3$

$7^x = 7^3$

$x = 3$

Solve: $9 = 27^x$ **need the bases to be the same**. BUT we cannot express *9 as a power of 27* **OR** *27 as a power of 9*. So, what base can we express 9 **and** 27 as a power of? $\Rightarrow$ 3

rewrite $9 = 3^2$ and $27 = 3^3$

$3^2 = (3^3)^x$ We have not changed the question, just the *look* of the question. Now, simplify the powers.

Your ***goal*** is to;

- Have a common base
- Single power on the left and right side of the equal sign
- Solve by setting the exponents equal to each other

Simplify (using laws of exponents) to create a single power.

$3^2 = 3^{3x}$ bases are now the same.....single powers .....now set exponents equal to each other

$2 = 3x$

$$\frac{2}{3} = \frac{3x}{3}$$

$$x = \frac{2}{3}$$ means that $9 = 27^{2/3}$

Solve: a) $4^{x+1} = 2^{x-1}$

get same base $(2^2)^{x+1} = 2^{x-1}$

$2^{2(x+1)} = 2^{x-1}$

use laws to get single power → $2^{2x+2} = 2^{x-1}$

∴ $2x + 2 = x - 1$

$2x - x = -1 - 2$

$x = -3$

b) $5^{2n+1} = \dfrac{1}{125}$

$5^{2n+1} = \dfrac{1}{5^3}$ same base but it is still the denominator

$5^{2n+1} = (5^3)^{-1}$ ← bring up to numerator by changing the sign of the exponent

$5^{2n+1} = 5^{-3}$

∴ $2n + 1 = -3$

$2n = -3 - 1$

$2n = -4$

$n = -2$

$\left(\dfrac{1}{5^3}\right)^1 = \left(\dfrac{5^3}{1}\right)^{-1}$

flip change sign

c) $4^{3x+1} = 32^x$ ← find a common base for 4 & 32

$(2^2)^{3x+1} = (2^5)^x$ $2^2 = 4$ $2^5 = 32$

$2^{2(3x+1)} = 2^{5x}$

$2^{6x+2} = 2^{5x}$ ← base are the same, single powers ∴ exponents are equal

∴ $6x + 2 = 5x$

$x = -2$

Solve: $5(2^x) = 80$

Remember….we want single powers on both sides of equal sign. Problem is on the left side….. two powers…. 5 and 2^x. Can we eliminate one of these powers???? Yes. Divide both sides by 5 (dividing by 2^x will create two powers on the right side)

$\dfrac{\cancel{5}(2^x)}{\cancel{5}} = \dfrac{80}{5}$

$2^x = 16$ — get same base

$2^x = 2^4$

$x = 4$

Solve: $65 = 2^{x+5} + 1$

Need single powers……move 1 to the other side.
Need same base

$64 = 2^{x+5}$

$2^6 = 2^{x+5}$ get same base

$6 = x + 5$

$x = 1$

A little more challenging now!!! ← see * at bottom for alternate method

Solve: $4^{x^2-x} = 4 \bullet 2^{x^2-x}$

Need single powers. The problem is on the right side. There is nothing that we can divide through by to create ***single powers.***

When there is multiplication, try to find a common base for ***all*** powers.

get common base

$$\left(2^2\right)^{x^2-x} = \underbrace{\left(2^2\right)\left(2^{x^2-x}\right)}_{\text{add exponents}}$$

On the right side, bases are the same with multiplication. Therefore you can use your exponent law…add exponents.

$$\left(2^2\right)^{x^2-x} = 2^{2+x^2-x}$$

multiply exponents →

$$2^{2x^2-2x} = 2^{x^2-x+2}$$

now set exponents equal to each other

$$2x^2 - 2x = x^2 - x + 2$$

a quadratic equation……remember how to solve a quadratic equation???????????

$$x^2 - x - 2 = 0$$

get zero on the right side, factor the left side if possible. (or use the quadratic formula)

$$(x - 2)(x + 1) = 0$$

$\therefore$ $x - 2 = 0$ or $x + 1 = 0$

$x = 2$ $\qquad$ $x = -1$

Homework worksheet 1.3 # 1-20 Challenge 22, 23

- everyone <u>should</u> attempt these
- Per 2 Gifted Cluster <u>must</u> do these.

* $\dfrac{4^{x^2-x}}{4^1} = \dfrac{4 \cdot 2^{x^2-x}}{4}$

$4^{(x^2-x)-1} = 2^{x^2-x}$

$(2^2)^{x^2-x-1} = 2^{x^2-x}$

$2^{2x^2-2x-2} = 2^{x^2-x}$

$\therefore 2x^2-2x-2 = x^2-x$

$x^2-x-2=0$

same as above after this

11 a) $2^{x+2} - 2^x = 48$

$2^x 2^2 - 2^x = 48$

$2^x(2^2 - 1) = 48$

$2^x(3) = 48$

$2^x = 16$

$2^x = 2^4$

$x = 4$