October 1st, 2008

2.7 Inverse of a Function $f^{-1}(x)$ "f inverse at x"

Determine the inverse of a function given:

- **Ordered Pairs**
  - interchange the x and y coordinates

$f(x) = \{(0, -2), (1,1), (2,4), (3, -2)\}$ function? yes

D = $\{0, 1, 2, 3\}$

R = $\{-2, 1, 4\}$

Invariant

$f^{-1}(x) = \{(-2,0)(1,1)(4,2)(-2,3)\}$ function? NO

∴ Inverses may not be functions

D = $\{-2, 1, 4\}$

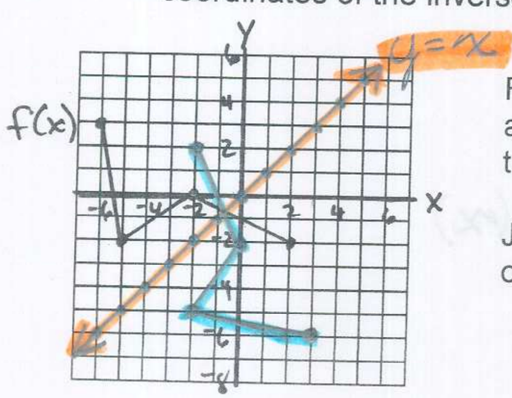
R = $\{0, 1, 2, 3\}$

** Domain of $f(x)$ is the Range of $f^{-1}(x)$

Range of $f(x)$ is the Domain of $f^{-1}(x)$

** **Invariant points** are points that are unchanged from the original function to the inverse function. These are points whose x and y coordinates are the same

- **Graph**
  - interchange the x and y coordinates of a given function to obtain the coordinates of the inverse.



Find the coordinates of all the **turning points** and interchange the coordinates of each point to obtain the new turning points

Join the new turning points in the correct order.

$f(x)$	$f^{-1}(x)$
(-6, 3)	(3, -6)
(-5, -2)	(-2, -5)
(-2, 0)	(0, -2)
(2, -2)	(-2, 2)

y=x is the axis of reflection

- **Equations**
  - interchange x and y variables in the equation and solve for y $\Rightarrow$ then rename y with y^{-1}.

Ex. $f(x) = 2x - 7$, find

y

a) $f(3)$

$f(3) = 2(3) - 7$
$= 6 - 7$
$= -1$

b) $f^{-1}(x)$

$x = 2y - 7$
$x + 7 = 2y$
$y^{-1} = \frac{x+7}{2}$
$f^{-1}(x) = \frac{x+7}{2}$

c) $f^{-1}(13)$

$f^{-1}(13) = \frac{13+7}{2}$
$= \frac{20}{2}$
$= 10$

$(13, 10) \rightarrow f^{-1}(x)$
$(10, 13) \rightarrow f(x)$

Inverse of Linear Functions

Determine the inverse of

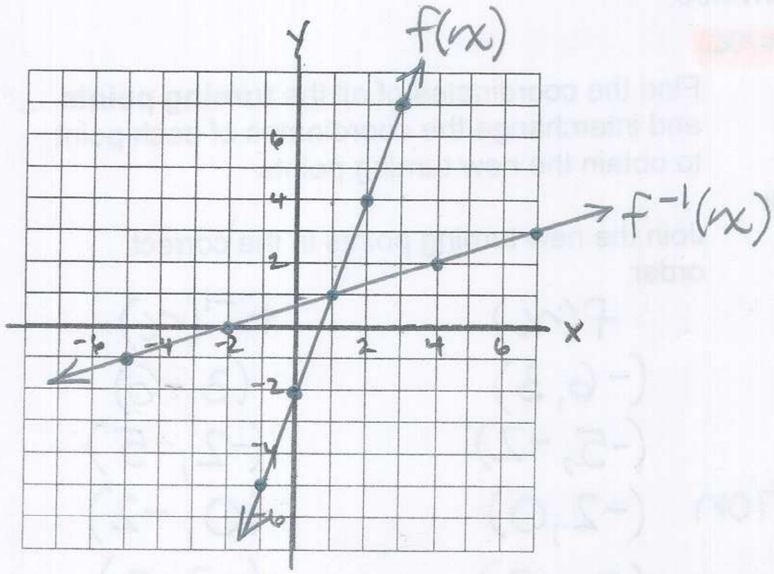
a) $f(x) = 3x - 2$ and graph

slope $\frac{3}{1}$
y vert -2
y

$x = 3y - 2$
$x + 2 = 3y$
$y^{-1} = \frac{x+2}{3}$

b) $g(x) = \frac{2}{3}x + 5$

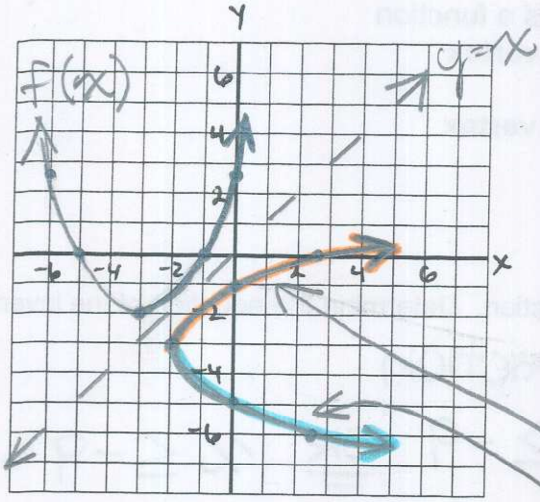
$x = \frac{2}{3}y + 5$
$x - 5 = \frac{2}{3}y$
$\frac{3}{2}(x - 5) = y$
$y^{-1} = \frac{3}{2}(x - 5)$



Inverse of Quadratic Functions

Determine the inverse of the function $f(x) = \frac{1}{2}(x+3)^2 - 2$

$$x = \frac{1}{2}(y+3)^2 - 2$$



$$2(x+2) = (y+3)^2$$

$$\pm\sqrt{2(x+2)} = \sqrt{(y+3)^2}$$

$$y+3 = \pm\sqrt{2(x+2)}$$

$$y^{-1} = \pm\sqrt{2(x+2)} - 3$$

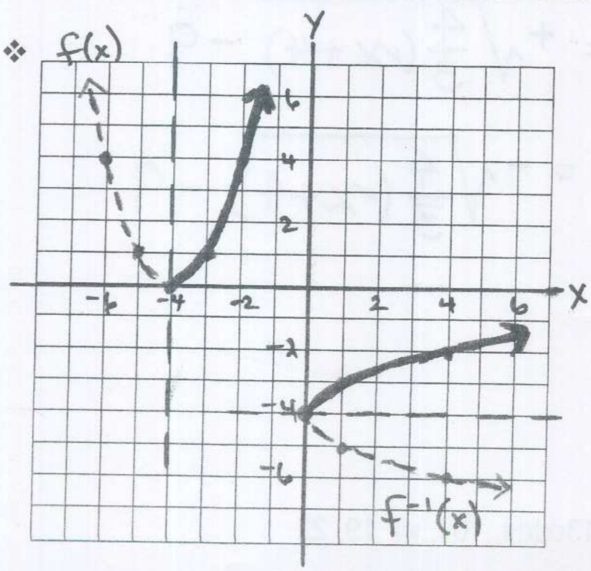
$$y^{-1} = +\sqrt{2(x+2)} - 3$$

$$y^{-1} = -\sqrt{2(x+2)} - 3$$

graph vertex $(-3, -2)$

step $\frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots$

- The inverse of a quadratic function is not a function.
- We can **restrict the domain** of the original function so that the inverse is a function.
-



For f(x) where $D = \{x \in \mathbb{R} \mid x \geq -4\}$ we get the right hand (Bold) half of the parabola and its inverse is $y^{-1} = +\sqrt{}$

For f(x) where $D = \{x \in \mathbb{R} \mid x \leq -4\}$ We get the left hand (dotted) half of the parabola and its inverse is $y^{-1} = -\sqrt{}$

Restrictions:

For the inverse of $f(x) = a[b(x-h)]^2 + k$ to be a function only one half of the f(x) can be used, either the left side of the axis of symmetry or the right side of the axis of symmetry.

To restrict the domain of f(x) so that $f^{-1}(x)$ is a function

Either $x \geq$ **x coordinate of the vertex**

Or $x \leq$ **x coordinate of the vertex**

Ex. Restrict the domain of g(x) so its inverse is a function. Determine the equation of the inverse.

$g(x) = \frac{3}{4}(x+9)^2 - 4$

Restriction

$x \geq -9$ OR $x \leq -9$

$x = \frac{3}{4}(y+9)^2 - 4$

$\frac{4}{3}(x+4) = (y+9)^2$

$y + 9 = \pm\sqrt{\frac{4}{3}(x+4)}$

$y^{-1} = \pm\sqrt{\frac{4}{3}(x+4)} - 9$

if $x \geq -9$ $\quad g^{-1}(x) = +\sqrt{\frac{4}{3}(x+4)} - 9$

OR if $x \leq -9$ $\quad g^{-1}(x) = -\sqrt{\frac{4}{3}(x+4)} - 9$

Homework page 215 #1-3 b each, 6cf, 10 i, iv, vi, 13odds, 16 i, vii, 19, 22